

Topic 7: Rules of Exponents

for use after *Comparing and Scaling* **Investigation 4**

Recall that you can use exponents to show repeated multiplication. For example, you can write $3 \times 3 \times 3 \times 3$ as 3^4 . You can also express repeated multiplication of rational numbers using exponents.

$$\begin{array}{ccc} \text{exponent} & & \text{value of the expression} \\ \downarrow & & \downarrow \\ \left(\frac{1}{2}\right)^5 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{32} \\ \underbrace{\hspace{1.5cm}} & \underbrace{\hspace{3.5cm}} & \\ \text{base} & \text{The base is used as a factor 5 times.} & \end{array}$$

Example 7.1

- A.** Write the expression $\frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5}$ using an exponent.

$$\frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} \times \frac{2}{5} = \left(\frac{2}{5}\right)^4$$

- B.** Find the value of the expression $(0.5)^3$.

$$(0.5)^3 = 0.5 \times 0.5 \times 0.5 = 0.125$$

Exercises

For Exercises 1–3, write each expression using an exponent.

1. $3.6 \times 3.6 \times 3.6 \times 3.6$ 2. $\frac{4}{9} \times \frac{4}{9} \times \frac{4}{9}$ 3. $0.8 \times 0.8 \times 0.8 \times 0.8 \times 0.8$

For Exercises 4–9, find the value of each expression.

4. $\left(\frac{1}{3}\right)^5$ 5. $(3.6)^2$ 6. $(0.4)^3$
7. $(2.1)^3 \times (3.5)^2$ 8. $\left(\frac{1}{5}\right)^3 \times \left(\frac{2}{3}\right)^2$ 9. $(0.2)^4 \times \left(\frac{3}{2}\right)^2$

10. For parts a–d, find the value of each expression.

a. $(0.1)^1$ b. $(0.1)^2$ c. $(0.1)^3$ d. $(0.1)^4$

- e. Describe the pattern in parts a–d for the exponent and the value of the corresponding expression.

- f. Use the pattern you described to write the value of $(0.1)^8$.

The rules of exponents allow you to rewrite some expressions. You can simplify an expression like $4^2 \times 4^3$, because 4^2 and 4^3 have the same base.

$$4^2 \times 4^3 = (4 \times 4)(4 \times 4 \times 4) = 4 \times 4 \times 4 \times 4 \times 4 = 4^5$$

Notice that $2 + 3 = 5$. To multiply two numbers with the same base, add the exponents.

You can use the rule for multiplying two numbers with same base to simplify an expression like $(6^5)^2$.

$$(6^5)^2 = 6^5 \times 6^5 = 6^{(5+5)} = 6^{10}$$

Notice that $5 \times 2 = 10$. To raise a power to a power, multiply the exponents.

Example 7.2

- A.** Write the expression $7^5 \times 7$ using a single exponent.

$$7^5 \times 7 = 7^5 \times 7^1 = 7^{(5+1)} = 7^6$$

- B.** Write the expression $(12^5)^{11}$ using a single exponent.

$$(12^5)^{11} = 12^{(5 \cdot 11)} = 12^{55}$$

Exercises

For Exercises 1–6, write each expression using a single exponent.

1. $7^2 \times 7^{13}$
2. $(3^6)^7$
3. $235^{141} \times 235^{37}$
4. $(17^4)^{20}$
5. $5^3 \times 5^9 \times 5^{16}$
6. $((2^4)^2)^5$
7. You can also simplify an expression like $\frac{9^5}{9^3}$, because 9^5 and 9^3 have the same base.
 - a. Rewrite $\frac{9^5}{9^3}$ using products instead of exponents in the numerator and denominator.
 - b. Simplify your expression from part (a) by dividing out the common factors of 9 in the numerator and denominator.
 - c. Rewrite your expression from part (b) using a single exponent.
 - d. Copy and complete the equation: $\frac{9^5}{9^3} = 9^{(5-3)} = 9^{\square}$.
 - e. Write a rule for dividing nonzero numbers with the same base.
 - f. Use your rule to write the expression $\frac{11^9}{11^2}$ using a single exponent.

The rules of exponents also apply to expressions with variables. For Exercises 8–10, write each expression using a single exponent.

8. $(x^5)(x^7)$
9. $\frac{m^{13}}{m^6}$
10. $(p^8)^3$

Topic 7: Rules of Exponents

PACING 2 days

Mathematical Goals

- Write and simplify expressions with rational numbers and exponents

Teaching Guide

The process for finding powers of rational numbers is the same as the process for finding powers of whole numbers. In Topic 7, students will practice writing expressions involving repeated multiplication using exponents. Students will simplify both decimals and fractions raised to powers. Students will also explore the rules of exponents.

After Example 7.1, have students find the value of fractions raised to a power by finding the products of the fractions. Show that raising the numerators and denominators to the power gives the same result.

Summarize the example by asking:

- *How can you write repeated products of two different numbers, such as $1.2 \times 1.2 \times 1.2 \times 7.5 \times 7.5$, using exponents?*
- *What is another way to write $\left(\frac{1}{2}\right)^4 \times \frac{1}{2}$?*
- *What is another way to write $\left(\frac{1}{2}\right)^4 \times \left(\frac{1}{2}\right)^2$?*

Homework Check

When reviewing Exercise 10, ask:

- *What is the value of 1 raised to any power?*
- *For a base that is greater than 1, does the value of a power increase or decrease as the exponent increases?*
- *For a base that is less than 1 but greater than 0, does the value of a power increase or decrease as the exponent increases?*

Before Example 7.2, explain to students that an expression with a base and an exponent, such as 2^5 , is called a power. An expression like $2^5 \cdot 2^6$ is a product of two powers. And expression like $(2^5)^6$ is a power raised to a power.

Summarize the example by asking:

- *Can you use the rule for multiplying two powers to simplify the expression $\left(\frac{1}{2}\right)^5 \times \left(\frac{1}{2}\right)^8$?*
- *Can you use the rule for multiplying two powers to simplify the expression $4^2 \times 5^2$?*

Before assigning Exercise 7, show students how to cancel common factors in the numerator and denominator of a fraction. Students may find it helpful to cross out common factors and write a 1 in the factor's place. Remind students that they must cancel the same number of common factors in the numerator as they do in the denominator.

Homework Check

When reviewing Exercise 7, ask:

- Can you use the rule for dividing two powers to simplify the expression $\frac{0.25^8}{0.25^4}$?
- Can you use the rule for dividing two powers to simplify the expression $\frac{5^9}{6}$?

Assignment Guide for Topic 7

Core Example 7.1 1–9, Example 7.2 1–6

Advanced Example 7.1 10, Example 7.2 7–10

Answers to Topic 7

Exercises for Example 7.1

1. $(3.6)^4$
2. $\left(\frac{4}{9}\right)^3$
3. $(0.8)^5$
4. $\frac{1}{243}$
5. 12.96
6. 0.064
7. 113.44725
8. $\frac{4}{1125}$
9. $\frac{9}{2500}$ or 0.0036
10. a. 0.1
b. 0.01
c. 0.001
d. 0.0001
e. Answers may vary. Sample: The 1 is the same number of decimal places to the right of the decimal point as the exponent.
f. 0.00000001

Exercises for Example 7.2

1. 7^{15}
2. 3^{42}
3. 235^{178}
4. 17^{80}
5. 5^{28}
6. 2^{40}
7. a. $\frac{9 \times 9 \times 9 \times 9 \times 9}{9 \times 9 \times 9}$
b. $\frac{9 \times 9}{1}$
c. 9^2
d. $\frac{9^5}{9^3} = 9^{(5-3)} = 9^2$
e. To divide two numbers with the same base, subtract the exponents.
f. 11^7
8. x^{12}
9. m^7
10. p^{24}